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Remnants from fast liquid withdrawal

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We study the breakup of an axisymmetric low viscosity liquid volume (ethanol and water), held by surface tension on supporting rods, when subject to a vigorous axial stretching. One of the rods is promptly set into a fast axial motion, either with constant acceleration, or constant velocity, and we aim at describing the remnant mass m adhering to it. A thin ligament is withdrawn from the initial liquid volume, which eventually breaks up at time t_b. We find that the breakup time and entrained mass are related by $t_b \sim \sqrt{m/\sigma}$, where σ is the liquid surface tension. For a constant acceleration γ, and although the overall process is driven by surface tension, t_b is found to be independent of σ, while m is inversely proportional to γ. We measure and derive the corresponding scaling laws in the case of constant velocity too.
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Since the seminal observations on the dynamics of liquid columns by Savart[1] and Plateau,[2] followed by the understanding of their stability and ultimate breakup by Rayleigh[3] and Eggers,[4] liquid jets, bridges, and ligaments have been recognized as the central objects mediating the formation of drops in Nature.[5,6] Liquid ligaments are often stretched axially when they form, either because they are accelerated by body forces like gravity,[7,8] or by surface stresses like a fast wind in various geophysical and industrial applications,[5,9] or because they have been impulsively accelerated on purpose like in inkjet printing[10] and some military explosives.[11] The impact of axial stretching on the stability of a liquid ligament has also a long history,[12,13] the lesson being that instability is suppressed as soon as the stretching rate overcomes the capillary instability rate based on the current ligament radius.[7,9,14,15] When formed by the rapid extension of a liquid bridge (a volume of liquid held by surface tension on supporting solid rods[16–19]), the ligament linking the distant pulling supports thus breaks up at its extremities,[20] and does so first in the region close to the solid where stretching vanishes.[8,9,21] (This is almost always true: the breakup can occur in other places for low stretching speeds.) As it stretches, the core of the ligament exchanges liquid with these dead zones, and the dynamics of the ligament volume is non-trivial.[8,22]

Yet, the question of the amount of liquid remaining adherent to the pulling rods has been eluded so far. Remnants adhering to a solid are relevant to all situations where an object immersed in a wetting liquid is quickly removed from it, and has obvious bearings on surface coating and cleaning,[23,24] animal feeding,[22,25] or metrology,[26–28] for example. This question has received answers when viscous stresses are balanced by capillarity in various forms of the Landau-Levich problem,[29] while we address here a situation where inertia and capillarity are solely at play: We study the breakup of an axisymmetric low viscosity liquid volume (ethanol and water), held by surface tension on supporting rods, when subject to a vigorous and reproducible axial stretching either at constant acceleration (up to $100g$, with $g = 9.81$ m/s^2), or constant velocity (up to 10 m/s).

Two slightly different experimental protocols are used: The first one, dedicated to the constant acceleration study, is composed of a shaft able to move axially smoothly with very little radial clearance, linked to a strong spring at its top [Fig. 1(a)]. The upper rod of radius R (0.5 mm $< R <$ 2.5 mm) is screwed to it at its bottom. Rods are made of different materials with different

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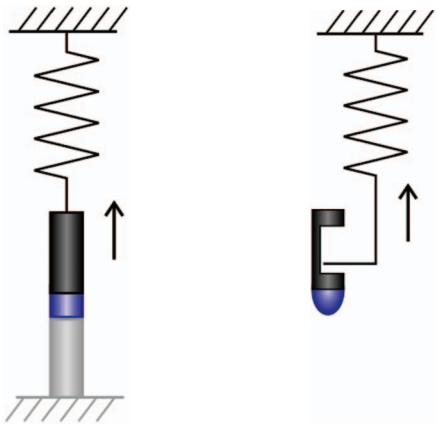

FIG. 1. (a) Constant acceleration experimental protocol: The spring, initially stretched, is released at $t = 0$, providing constant acceleration to the upper rod, while the lower rod (if any) stays still. (b) Constant velocity protocol: The spring is released and sets into motion a metal piece which, at $t = 0$, hits and propels the rod holding the drop at velocity U.

liquid/solid wettabilities: steel, aluminium, brass, and glass. The spring is locked at the desired extension, and when released, the upper rod is entrained at acceleration γ such that 10 m/s^2 $< \gamma <$ 1200 m/s^2.

The second protocol [Fig. 1(b)], dedicated to the constant velocity study, relies on the same *stretch-and-release* principle: A hollow metal capsule, holding the rod and droplet, is loosely hanged at the desired altitude. A heavy rigid metal piece, linked to the spring, is accelerated upwards when the latter is released, and eventually hits the capsule, thus propelled at velocity U such that 0.45 m/s $< U <$ 9.1 m/s.

A liquid drop is gently deposited on the upper rod, and squeezed by another identical facing rod (making a bridge), or not (leaving it as a pendant drop), as seen in Fig. 2. Liquid bridges have been stretched according to the constant acceleration protocol only. The two protocols are conducted with tap water and ethanol, which have similar viscosities η (1 mPa s and 1.2 mPa s, respectively), comparable densities ρ (1000 kg/m^3 and 789 kg/m^3), and very different surface tensions σ

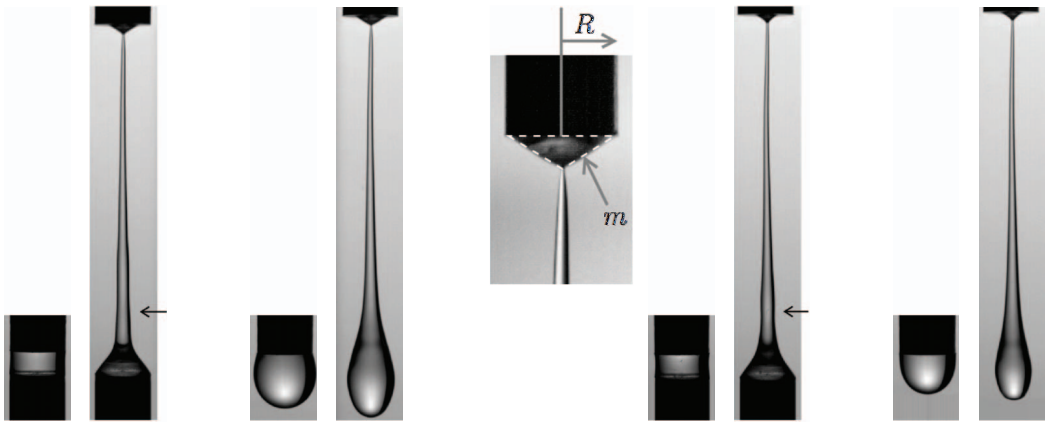

FIG. 2. Four stretching sequences for water (first four images) and ethanol (last four images). The acceleration of the upper rod ($\gamma = 57 \pm 1$ g) and the rod radii ($R = 1.50 \pm 0.01$ mm) are the same for all the experiments. For each experiment, the first image corresponds to the initial condition and the second image to the breakup time. The absolute breakup time $t_a = 10 \pm 0.1$ ms, from the onset of the motion, is the same for all the experiments, although the initial conditions and the surface tension σ are different. The horizontal black arrow indicates the height at which the bridge would be quasi-statically unstable. The middle inset defines the extracted mass m at breakup time, approximated by a conical volume, and the radius of the rod R.

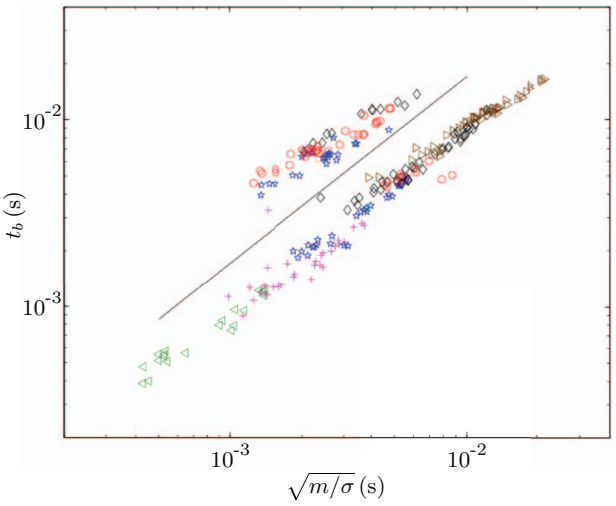

FIG. 3. Breakup time of the ligament as a function of $\sqrt{m/\sigma}$ (in seconds). Constant acceleration experiments (top set of data) and constant velocity (bottom), with **water**: $R = 0.5$ mm (green $\triangleleft$), 1 mm (magenta $+$), 1.5 mm (blue $\star$), and 2.5 mm ($\diamond$); with **ethanol**: $R = 1$ mm (orange $\square$), 1.5 mm (red $\circ$), and 2.5 mm (brown $\triangleright$). Data for $Bo < 7$ and $We < 15$ are not shown.

(65 $\pm$ 3 mN/m and 22.3 $\pm$ 1.0 mN/m). Depending on the protocol, the stretching strength is measured either by the acceleration-based Bond number $Bo = \rho\gamma R^2/\sigma$, or by the Weber number $We = \rho U^2 R/\sigma$. The Ohnesorge number $Oh = \eta/\sqrt{\rho\sigma R}$ typically ranges between 2.5×10^{-3} and 7.4×10^{-3}, so that viscous slowing is negligible.[15] The accelerations γ being much larger than the free-fall acceleration g, gravity is also negligible. The presumably inexistent effect of ambient air on the dynamics of the liquid is not considered either. The motion is recorded with a high-speed camera. The spatial resolution is such that the rod is typically 100–150 pixels wide. The frame rate of the video recordings was adjusted to retain the error for the breakup time low (<5%): typically around 8000 fps, and up to 20 000 fps for $R = 0.5$ mm. Moreover, all the events reported in this paper occur in a time range in which the acceleration or the velocity of the upper rod is constant, up to an error of a few percents.

Motions inside an initially still inviscid liquid ligament of radius R are ruled by the capillary timescale $\tau_c = \sqrt{\rho R^3/\sigma}$ setting, for instance, the order of magnitude of its breakup time.[3,15] The meaning of this result is transparent: The length scale in the axial direction which is mostly unstable is given by R, carrying a corresponding mass of order ρR^3. Its inertia is moved by surface tension forces hence, owing to Newton's law, the structure of τ_c. More generally, if m is the mass in the ligament moved axially to achieve breakup, then the breakup time is, for the same reason

$$t_b \sim \sqrt{m/\sigma}. \tag{1}$$

Note that the mass m represents the mass of fluid actually in motion. For instance, when withdrawing liquid from an ocean by a small pipette, the mass at play will be the one withdrawn by the pipette remaining attached to it, and not the mass of the ocean. Fig. 2 shows how we define breakup, namely, when the ligament looses its connectivity with the pulling rod, and Fig. 3 presents experimental data of the breakup time t_b as a function of $\sqrt{m/\sigma}$, the mass remaining attached to the upper rod, also defined in Fig. 2: the mass is computed as that of the conical volume adhering the rod at breakup. The error associated with this approximation is typically less than 10% and reaches a maximum of 20% for very small mass ($\sim 10^{-8}$ kg), for maximum magnification reasons. The error bars associated with time or length measurements, though not shown in Figs. 3–6, would be much smaller than the scattering of points in these figures, which gives a precise idea of the standard deviation of the physical process itself. The breakup time t_b is defined as the absolute time t_a when connectivity is lost from the start of the motion of the upper rod, minus the time needed to reach the height at which the ligament would be quasi-statically unstable t_i. The line in Fig. 3 represents the scaling law of

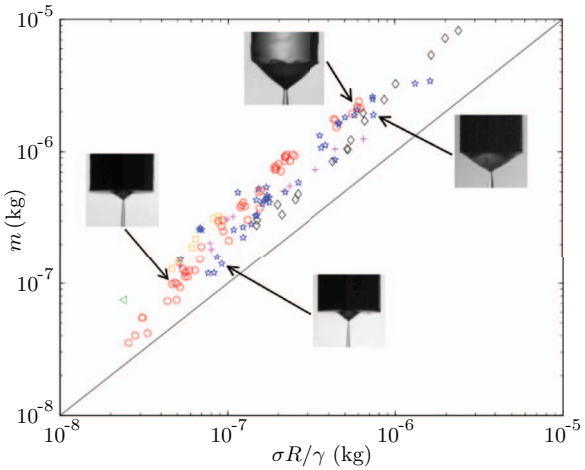

FIG. 4. Extracted mass as a function of $\sigma R/\gamma$ (in kg). The solid line is the scaling in Eq. (2) and the symbols are the same as in Fig. 3. The four insets correspond to the last instant before breakup, for: (left) ethanol, $R = 1.5$ mm, $\gamma = 66\,g$; (top) ethanol, $R = 1.5$ mm, $\gamma = 4\,g$; (right) water, $R = 1.5$ mm, $\gamma = 11\,g$; (bottom) water, $R = 1.5$ mm, $\gamma = 112\,g$.

Eq. (1), agreeing well with experiments – both at constant acceleration and constant speed – not only in law, but also in absolute value.

When the acceleration γ of the upper rod is imposed constant, a simple expression for the extracted mass m is readily derived: The only force pulling the fluid mass is the surface tension force, that is $f \sim \sigma R$ for a contact line pinned at the edge of the solid cylinder, up to a contact angle prefactor. Then Newton's law stating that $m = f/\gamma$ implies that

$$m \sim \sigma R/\gamma. \tag{2}$$

This is, in the frame of reference of the upper rod, the maximal mass of fluid pulled by a body force γ that surface tension can keep in equilibrium. Fig. 4, which also displays how the remnant mass looks like, shows that the experimental results are in good agreement with Eq. (2). Combining Eqs. (1) and (2), we expect the breakup time to be

$$t_b \sim \sqrt{R/\gamma}, \tag{3}$$

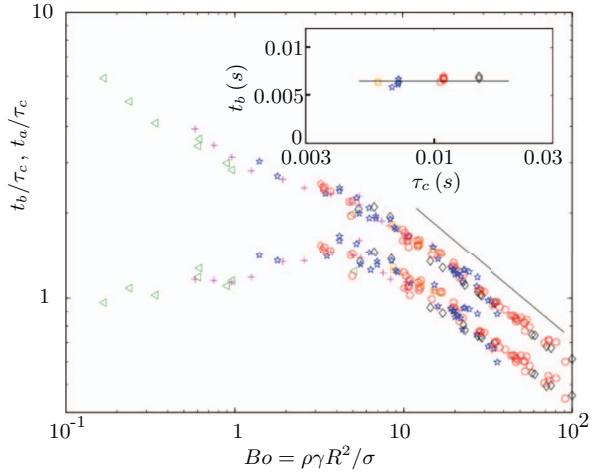

FIG. 5. Non-dimensional breakup time of the ligament from static instability threshold t_b/τ_c (bottom set of data) and non-dimensional absolute breakup time t_a/τ_c (top set of data), as a function of the Bond number. The symbols are the same as in Fig. 3. The solid line is the scaling law in Eq. (4). Inset: Dimensional constant breakup time for experiments with various R, σ, and ρ but fixed $\sqrt{R/\gamma} = 1.550 \pm 0.005$ ms.

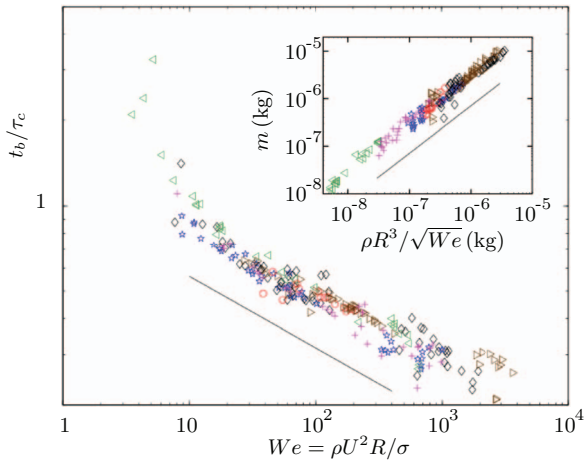

FIG. 6. Non-dimensional breakup time t_b/τ_c of the ligament as a function of the Weber number. The solid line is the scaling law in Eq. (7). Inset: Extracted mass m as a function of $\rho R^3/\sqrt{We}$ (in kg). The solid line is the scaling law in Eq. (6) and the symbols have the same meaning as in Fig. 3. Notice the scattering of points for large values of the Weber number: this is due to spurious vertical oscillations of the rod. Apart from this type of error, the experimental results are highly reproducible, both in breakup time and extracted mass.

a surprising result at first sight in which surface tension has disappeared; but surface tension is both responsible for pulling the liquid (Eq. (2)), and pinching it (Eq. (1)), causing this fortuitous compensation effect. Using the capillary timescale $\tau_c = \sqrt{\rho R^3/\sigma}$, the above scaling becomes

$$t_b/\tau_c \sim \sqrt{\sigma/\rho\gamma R^2} = Bo^{-1/2}. \tag{4}$$

Fig. 5 compares this scaling with experiments obtained with different values of R, σ, ρ, and γ, for liquid bridges and pendant drops, and different rod materials (steel, aluminium, brass, and glass). The absolute time to breakup t_a (top set of points) is plotted together with the breakup time t_b defined previously (bottom set of points). Stretching has little effect below $Bo \simeq 7$: in this range, the breakup time is slightly increasing but reliably given by the capillary time τ_c.[9] For $Bo \gtrsim 7$ however, stretching has a strong effect on breakup, and the scaling law in Eq. (4) represents well the experiments.

When a pendant drop is stretched upwards with a constant velocity U, Fig. 3 shows that Eq. (1) still holds for describing its breakup time. However, the scaling of the remnant mass m with respect to the physical parameters ρ, σ, R, and U, when it is prescribed by a simple force balance in the constant acceleration case, is now different. The axial displacement of the moving support mediates the motion of the liquid in the ligament attached to it via capillary waves. These are progressive waves of amplitude proportional to $e^{ikz - i\omega t}$ with wavenumber k and pulsation ω whose dynamics is well approximated by[15,30]

$$\omega^2 = -\frac{1}{2\tau_c^2}\left[(kR)^2 - (kR)^4\right], \tag{5}$$

which is also Newton's law of inertia, now expressed in its local version. Only the waves outside the unstable range (i.e., $kR > 1$, for which ω has a non-zero real part) are liable to propagate, with group velocity $\partial\omega/\partial k \approx kR^2/\tau_c$. Therefore, whatever the imposed driving velocity U, it is always possible to find a wavenumber within this stable range, that travels with this imposed speed. The entrained mass is thus proportional to $\rho R^2/k$, with k the wavenumber whose velocity equals that of the support U. We find $k \sim \sqrt{We}/R$ and thus

$$m \sim \rho R^3/\sqrt{We}, \tag{6}$$

a scaling for m which Fig. 6 confirms quantitatively. Using Eqs. (1) and (6), the dimensionless breakup time t_b/τ_c is expected to be

$$t_b/\tau_c \sim (\sigma/\rho R U^2)^{1/4} = We^{-1/4}, \tag{7}$$

a scaling law again in good agreement with experiments, as seen in Fig. 6. The breakup time when the pulling *velocity* is prescribed now depends intrinsically (albeit weakly) on surface tension, as opposed to the case in Eq. (3) where the *acceleration* is imposed.

How much liquid can be withdrawn from a rod dipping at the surface of a pool ? In answering this question, we have shown that surface tension mediates in all cases the stresses responsible for liquid entrainment, and that the time it takes to complete the operation is always related to the mass of the remnants by Eq. (1). However, we have also shown that the protocol matters and that not only the strength of the withdrawal counts, but also the nature of its motion: The breakup time is independent of liquid surface tension if the rod is withdrawn at constant acceleration; it is weakly dependent on it if withdrawal is made at constant velocity. In all cases, the mass of the remnants is decreasing with withdrawal strength, a fact known to anyone who has ever drained leaves of salad, or his wet hands.

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